where S is the slot width

We now calculate the total film flow rate W_{film} required to maintain $T_{w\,g} \leq T_a$ (i e , $\eta \geq \eta_a$) along a section of the duct of length l (perimeter b) Substituting $\eta = \eta_a$ and $x = l$ into Eq (12), one obtains for injection through a single slot at $x = 0$,

$$W_{\text{film}} = bw = bl\,\frac{h_g}{c_p}\,\frac{Re^{\,1/8}}{[0\ 04Re^{\,1/8} + \ln(1/\eta_a)]} \tag{15}$$

For injection through multiple slots with spacing Δx_j along the flow direction, Eq (12) determines the appropriate $\Delta w_{c\,j}$ required to maintain $\eta \leq \eta_a$ on the assumption that the interference effects in multiple slot injection are negligible Because of the linear relation between Δx_j and $\Delta w_{\ j}$, then, the total injection rate $W = \Sigma \Delta w_{c\,j}$ along the length $l = \Sigma \Delta x_j$ is the same as W_{film} in Eq (15) (Actually, $W \lesssim W_{\text{film}}$ owing to the beneficial effect of multiple slot interference [5])

Comparison of Total Flow Rates

To effect a comparison of the total flow rate requirements, we note the following relation between η_a and R_a which follows from the use of Eqs (2, 5, and 11) with $T = T^{\,0}$:

$$\eta_a = 1 - R_a/C \tag{16}$$

Substituting η_a in Eq (15) for W_{film} and using Eq (7) for W_{transp}, we obtain the ratio

$$\lambda = \frac{W_{\text{transp}}}{W_{\text{film}}} = 3\left(\frac{1}{R_a^{\,1/3}} - 1\right) Re_s^{\,-1/8} \times \left\{0\ 04\,Re^{\,1/8} + \ln\frac{C}{C - R_a}\right\} \tag{17}$$

The dependence of λ on T_a implicit in R_a is graphically shown in Figs 1 and 2 for injection Reynolds number $Re = 10^2$ and $Re = 10^4$, respectively, with C as a parameter It is seen from these curves that under certain conditions of convective cooling, $C < 1$, the total flow rates required in film cooling are smaller than those required in transpiration cooling, as defined by the criterion $\lambda > 1$ For the adiabatic wall ($C = 1$), λ is always less than unity except in the immediate neighborhood of $R_a \rightarrow 0 (T_a = T\ )$ The latter condition can be approached in transpiration cooling only for very large injection rates The occurrence of the asymptotes at $R_a \rightarrow C < 1$ is because of the diminishing film-injection requirements as $T_a \rightarrow T_{w\,g}{}^0$ Thus, the criterion $\lambda > 1$, corresponding to the regime in which film injection is advantageous, is generally satisfied when only marginal reduction of the wall temperature is required by coolant injection

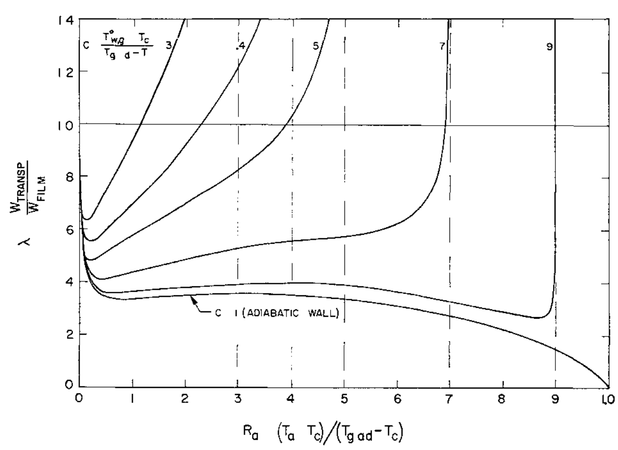

Fig 1 Comparison of transpiration and film coolant flow rates for convectively cooled walls ($Re_s = 10^2$)

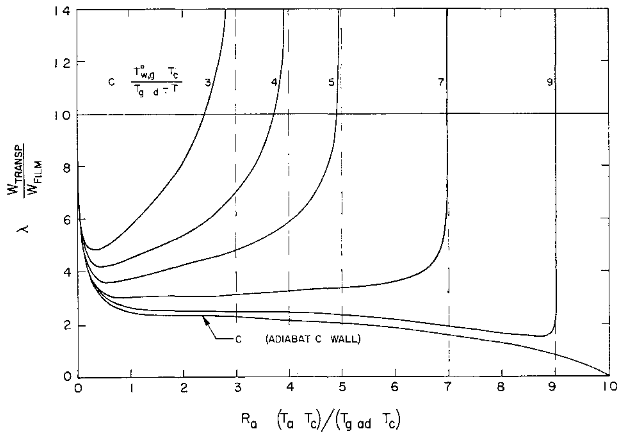

Fig 2 Comparison of transpiration and film coolant flow rates for convectively cooled walls ($Re = 10^4$)

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Maximum Rendezvous Launch Window Characteristics

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Nomenclature

$H\,A$ = hour angle
i = target inclination
i_R = relative inclination
LW = launch window
t_w = wait time
θ_0 = equatorial reference angle
ϕ_L = launch site latitude
ψ = launch azimuth, measured from north
ω = rotational velocity of earth

LAUNCH hold delays, azimuth constraints, and phasing considerations normally preclude the ground launch of a rendezvous resupply vehicle into the plane of a target orbit Ground waiting and/or orbit waiting is therefore a necessary prerequisite to the rendezvous maneuver [1 2]

It is the purpose of this note to define a target orbit inclination that provides the maximum uninterrupted launch window for any particular launch latitude Launch window,

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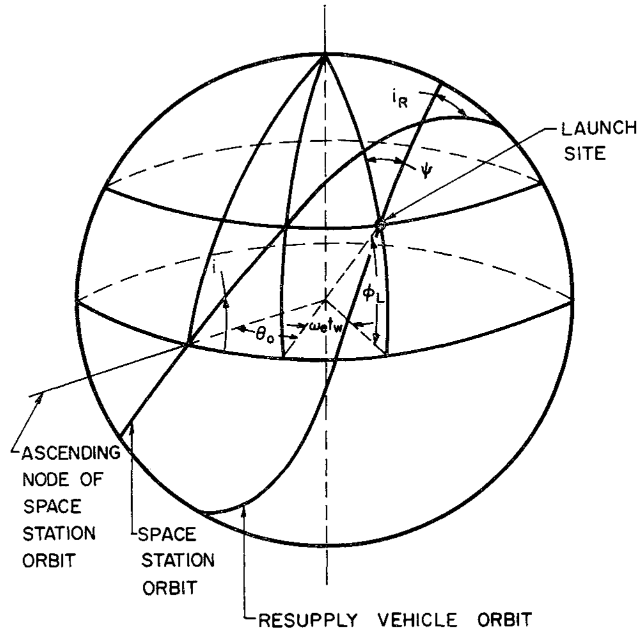

Fig 1 Coordinate schematic

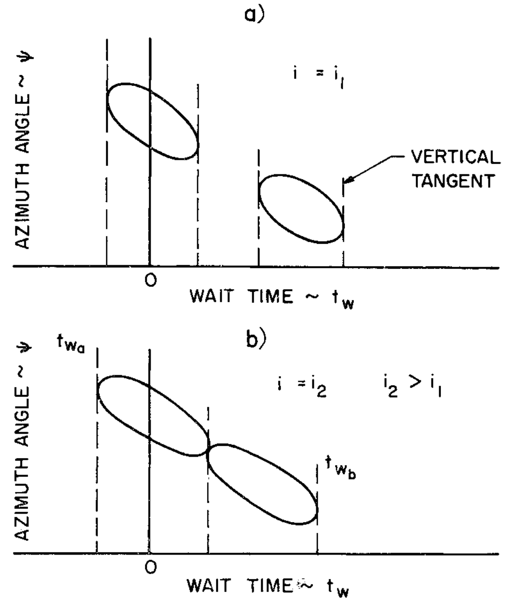

Fig 2 Graphical relation between azimuth angle and wait time

as defined herein, is that available launch time when the relative inclination between the target and resupply orbits is less than a specified value (i_R)

The following analysis is based on geometric rather than dynamic considerations Perturbative forces that act on the resupply vehicle and the warping of the resupply launch plane during the powered ascent are known and can be compensated for [3]

A geometric schematic of the maneuver is presented in Fig 1 Wait time is arbitrarily referenced to the instant when the launch site at launch is contained in the plane of the target This situation exists only if the launch site latitude (ϕ_L) is less than the target orbit inclination If $\phi_L > i$, then zero time is defined when the launch site and maximum target latitude lie along the same meridian Negative wait time is equivalent to launch site positions at launch prior to the launch site-target plane intersection Positive time is counted when the earth's rotation carries the launch site past the initial target plane intersection position

Based on the foregoing wait time definition, the hour angle ($H A$) measured in the plane of the equator from the target ascending node to the launch site at any instant is

$$H A = \theta_0 + \omega t_w$$

where

$$\sin\theta_0 = \tan\phi_L/\tan i \qquad \phi_L < i$$
$$\theta_0 = \pi/2 \qquad \phi_L \geq i$$

By appropriate axis rotations, the relative inclination is given by

$$\cos i_R = \sin\phi_L \sin i \sin(\theta_0 + \omega t_w) \sin\psi + \sin i \cos(\theta_0 + \omega t_w) \cos\psi + \cos\phi_L \cos i \sin\psi \quad (1)$$

Solving Eq (1) for the launch azimuth angle (ψ) gives

$$\psi = \text{arc} \sin[a^2 + b^2]^{-1/2} - \text{arc} \tan[b/a] \quad (2)$$

where

$$a = \frac{\sin\phi_L \sin(\theta_0 + \omega_e t_w) + \cos\phi_L \cos i}{\cos i_R}$$

$$b = \frac{\cos(\theta_0 + \omega_e t_w)}{\cos i_R}$$

The fundamental relation between azimuth angle and wait time has the property of the leminscate [4] When $i > \phi_L > 0$, and for a small fixed relative inclination, the function ψ vs t_w describes two closed curves (ovals of Cassini; see Fig 2a)

As the target inclination decreases, the curves coalesce When the ovals are tangent (see Fig 2b), ψ is a single-valued parameter for three values of t_w This condition corresponds to the maximum target inclination that allows a continuous launch window from t_w to t_{w_b} (Fig 2b) It is therefore required to find the target inclination that causes this coalescence

In order to obtain single-valued solutions for ψ, the arc sin term in Eq (2) must equal $\pm\pi/2$ or $a^2 + b^2 = 1$ Substitution and simplification of this relation yields

$$\sin(\theta_0 + \omega t_w) = \frac{\sin\phi_L \cos i \pm \sin i_R}{\cos\phi_L \sin i} \quad (3)$$

Equation (3) indicates that there may be four values of (t_w) which yield a single value for ψ As shown in Fig 2a, these values of t_w define the position of two exterior and two interior vertical tangents

The numerical values of t_w which define the position of the interior tangents result from the positive sign preceding

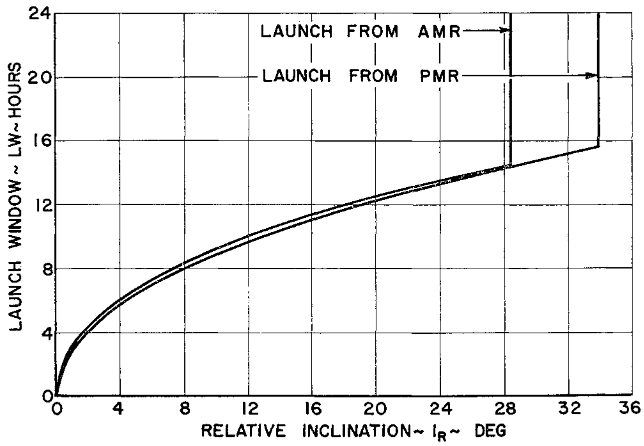

Fig 3 Maximum continuous launch window characteristics for AMR and PMR

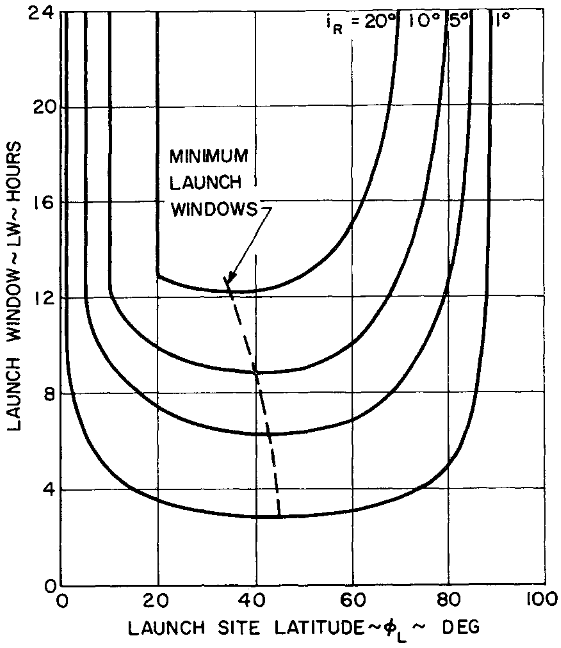

Fig 4 The effect of launch site latitude on the maximum continuous launch window characteristics

$\sin i_R$ The interior tangents merge or the ovals coalese when $\sin(\theta_0 + \omega t_w) = 1$; hence,

$$1 = \frac{\sin\phi_L \cos i + \sin i_R}{\cos\phi_L \sin i} \quad \text{or} \quad i = i_R + \phi_L \qquad (4)$$

When Eq (4) is satisfied, the maximum continuous launch window is defined by the exterior tangents $(t_{wa} - t_{wb})$ In this instance, the maximum launch window is

$$LW = (2/\omega)[(\pi/2) - \theta_0 - \omega t_w] \qquad (5)$$

The relation between launch window, launch latitude, and relative inclination is obtained by substitution of Eqs (4) and (5) into (3) This yields

$$LW = \frac{2}{\omega} \text{ arc cos}\left[\frac{2x \tan\phi_L}{\tan(\phi_L + i_R)} - 1\right] \qquad (6)$$

The relation between the maximum launch window and relative inclination for powered ascents from Atlantic Missile Range (AMR) and Pacific Missile Range (PMR) are given in Fig 3 In the absence of launch azimuth constraints, the AMR launch window exceeds the PMR launch window for any given relative inclination The vertical lines shown represent an unrestricted launch window capability (LW = 24 hr) Launch at any time can be achieved, when the relative inclination exceeds the launch site latitude, by injecting the target into an equatorial orbit This method is not consistent with minimum propellant requirements

The variation of launch window with launch site latitude for relative inclinations of interest is presented in Fig 4

Maximum launch windows will exist for launch sites located in the region of the equator or poles

The launch latitude corresponding to the minimum launch window depends on the relative inclination and is represented by the dotted line The locus of minimum launch windows is defined by $dLW/d\phi_L = 0$ and is given by the relationship

$$\phi_{L_{\min}} = \text{arc tan}[(1 - \sin i_R/\cos i_R]$$

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Lift Contribution to the Sonic Boom

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When making a theoretical estimate of the sonic boom from a supersonic aircraft or missile, it is customary to separate the effects of volume and lift In doing this, most workers (e g, Refs 1 and 2) follow Walkden[3] and assume that, in any given azimuthal plane, the boom can be represented as that of an equivalent body of revolution Walkden's work is based, in turn, on that of Whitham,[4] who makes the key assumption that "the propagation of the disturbance down each ray tube may be treated separately" However, the strength of the wave from a lifting body will vary around the azimuth, and there will be a tendency for this variation to be evened out by waves propagating in the circumferential direction Being three-dimensional in nature, this effect would be very difficult to incorporate in an analytical approach, and some experiments have been performed to gain some information on the possible significance of this "circumferential evening" effect

The experiments were carried out in the William and Agnes Bennett Supersonics Laboratory in the University of Sydney, using a series of sting-mounted models in the 5- × 5-in tunnel operating at a Mach number of 2 The models could be rotated about an axis parallel to the stream and were adjustable in the vertical plane The bottom wall of the tunnel was provided with a series of pressure holes that enabled the pressure change across the reflected bow shock to be measured The waves were sufficiently weak for this pressure change to be very nearly twice the pressure difference across the bow shock The general layout of the experiment is shown in Fig 1, which also defines the notation

Figure 2 shows a comparison of the shock strength from a 15° semiapex angle cone at zero incidence and at an incidence of 15° It is seen that the wave from the cone at incidence becomes progressively more uniform around its periphery At the maximum distance at which it was possible to take measurements, its strength was fairly uniform with the azimuth angle θ and was not very different from the strength of the wave from the corresponding cone at zero incidence

Rhyming and Yoler[1] have specified the family of flat delta wings with subsonic edges which conventional theory indicates is equivalent to a cone at zero incidence Figure 3 compares the measured shock strengths from a wing that is predicted to be equivalent to the 15° semiapex angle cone with those from the cone Over the range of distances at which measurements could be taken, the maximum pressure from the wing is only slightly higher than that from the cone (This would be expected to be high, since the actual wing necessarily has some thickness) However, a progressive evening up of the shock strength around the pe-

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